ENGINEERING MATHEMATICS III

(Based On SASTRA UNIVERSITY syllabus)

Question Bank

CHAPTER 1 – LAPLACE TRANSFORM.

CHAPTER 2 – COMPLEX DIFFERENTIATION.

CHAPTER 3 – COMPLEX INTEGRATION.

CHAPTER 4 – FOURIER TRANSFORM.

BY,

CONNECT SASTRA TEAM.

CHAPTER – 1 – LAPLACE TRANSFORM.

<u>2 Marks :</u>

(5 two marks will be asked from this model)

1 .Define Laplace Transform .

2. State the condition for the existence of Laplace Transform.

3. Define the Laplace transform of Unit step Function.

4. Define the Laplace transform of Unit Impulse Function.

5. State Linear Property of Laplace Transform. *

6. State First Shifting Property of Laplace Transform. *

7. State Second Shifting Property of Laplace Transform. *

8. State Convolution theorem of Laplace Transform.

9. State Initial & Final value theorem of Laplace Transform. *

10. Define Laplace transform of derivatives. *

11. Define the Laplace transform of Integration. *

12. Two Marks Problem Using * (above star ‘ ed) theorem & Property.

13. Simple Problem Using Laplace Transform & Inverse Laplace Transform.

14. Find the Laplace transforms of

(a) $\sin 2t \sin 3t$
(b) $\cos^2 2t$
(c) $\sin^3 2t$
(d) $e^{-3t} 2\cos 5t - 3\sin 5t$
(e) $e^{3t} \sin^2 t$
(f) $t \cos at$
(g) $t^2 \sin at$
(h) $\frac{1-e^t}{t}$
(i) $t^3 e^{-3t}$
(j) $te^{-t} \sin 3t$
(k) $\sin kt - kt \cos kt$
(l) $\frac{\sin at}{t}$
(m) $\sin^2 t$

15. Write a function for which Laplace transformation does not exist. Explain why Laplace transform does not exist.
16. If L(f(t)) = F(s) what is L(e^{-at}f(t))?
17. Find the Laplace transform of $e^{-2t}(1+t)^2$.
18. Find the Laplace transform of te^{-t} sint.
19. Find L(tsin2t).
20. Obtain the Laplace transform of sin2t-2tcos2t in the simplified form.
21. Verify the initial value theorem for f(t) = 5 + 4cos2t.
22.Find the Laplace transform of t coshat.
23.Find the Laplace Transform of unit step function at t=a.
24.Does the Laplace transform of Cos a t/ t exist? Justify.
25.Find *L^{-1} {(cot^{-1} (s) }*

15 marks :

(You can Omit Atmost one Part Mentioned Below)

PART 1 – FIND THE LAPLACE TRANSFORM OF _______.

PART 2 – FIND THE LAPLACE INVERSE OF ______.

PART 3 – LAPLACE TRANSFORM OF PERIODIC FUNCTIONS.

PART 4- LAPLACE INVERSE USING CONVOLUTION THEOREM.

PART 5 – APPLICATION TYPE (LAPLACE TRANSFORM OF DIFFERENTIATION & INTEGRATION i.e SOLVE TYPE SUM)

CHAPTER 2 – COMPLEX DIFFERENTIATION

2marks :

1. Show that _____ is analytic.

2. Show that _______ is harmonic.

3. Test the analyticity for _____.

4. Check Whether ______ can be real part (or) imaginary part of an analytic function.

5. Find the Critical points for ________.

6. Find the invariant Points (OR) Fixed points of ______.

7. Find f(z) if real part (OR) imaginary part is given as _____.

8. Find a , b , c if f(z) = ______ to be analytic.

(Note : " _____ " implies that model problem will come ")

9. Show that analytic function with Constant Real part (OR) Constant imaginary Part (OR) Constant modulus (Argument) part is constant.

10. State the properties of analytic function.

11. State necessary & sufficient conditions for f(z) to be analytic.

12. Define Analytic (or) Regular (or) Holomorphic function.

13. Write C.R equations in Cartesian form (OR) Polar Form.

14. Define Bilinear (OR) Mobius Transformation.

15. Define Confirmal Mapping.

16. Define Critical point.

17. Define Singular Point.

18. Define Invariant Point (OR) Fixed Point.

19. Define Isogonal Transformation.

20. Define Cross ratio.

21. If u , v are analytic , prove that u+iv is also analytic.

22. If u , v are harmonic , prove that u+iv is also analytic.

23. If u+iv is analytic , prove that v-iu is also analytic.

24.If f(z) is analytic , prove that k x f(z) is also analytic. (where k is constant).

25. What is the necessary condition for the existence of the derivative of f(z)?

26. If w = log z, then determine where w is non analytic.

27. Define singular point of the function with an example.

28. Show that f(z) = xy + iy is not analytic.

29. Define holomorphic function.

30. Show that v = e^x sin y is harmonic function.

31. Show that v = e^x cos y is harmonic function.

32. Find the invariant points of the transformation w = $z-1/z+1$.

33. Under the transformation w = $1/z$, find the image of |z − 2i| = 2.

34. Show that the transformation w = $1/z$ transforms all circles and straight lines to circles and straight lines in the w-plane.

35. State two important properties of Mobius transformation.

36.Prove that the real and imaginary part of an analytic function satisfies Laplace equations.

37. Is f (z) = z^3analytic ? Justify.

38. Prove that $z z$ is nowhere analytic.

39 .For what values of a,b and c the function $f(z) = x - 2ay + i(bx - cy)$ is analytic.

40. If u+iv is analytic , show that v – iu &-v + iu are also analytic.

41. State the orthogonal property of an analytical function.

42. Write down the formula for finding an analytic function f (z) = u + iv, whenever the real part is given by using Milne Thomson method.

43. Find ‘a’ so that u (x,y) = $ax^2 - y^2 + xy$ is harmonic.

(Note : If they Ask , Prove that log z is analytic , at last we have to write “log z is analytic except at z=0)

15 marks :

(You can Omit Atmost one Part Mentioned Below)

PART 1 – Find the analytic function whose u (OR) v (OR) u+v (OR) u-v is given as

PART 2 – Confirmal Mapping.

PART 3 – Bilinear Transformation

PART 4 - Proof Sum.

PART 5 – Image of Transformation Sum .

Part 6 - Problem Using Milne Thomson Method.

Proof Sums:

1. If $f(z)$ is a regular function of z prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$.
2. If $f(z)$ is an analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log|f(z)| = 0$.

3.Prove that “If v is a harmonic conjugate of u, then the two families of curves $u(x, y) = \alpha$ and $v(x, y) = \beta$ are mutually orthogonal to each other’’.

CHAPTER 3 – COMPLEX INTEGRATION .

2marks :

1. State Cauchy's Integral theorem (OR) Cauchy's Fundamental theorem .
2. State Cauchy's Integral theorem Formula.
3. State Cauchy's Integral Formula for derivatives.
4. Simple Problems using Cauchy's Integral Formula.
5. State Taylor's series.
6. State Laurent's Series.
7. Simple Problems using Taylor's & Laurent Series.
8. Define Singular Point.
9. Define Isolated Singularity . Give an example.
10. Define Essential Singularity. Give an example.
11. Define Removable Singularity. Give an example.
12. Define Residue at a Pole.
13. Define Pole & its type.
14. State Cauchy's Residue Theorem.
15. Simple Problems Using Cauchy's Residue Theorem.
16. What is analytic & Principal part of Laurent's Series.
17. What are the poles of cot z ?
18. Define Mesomorphic Function.
19. Define Simply & multiply Connected Region.
20. What are the formula's to find the residue of a function at a simple pole ?
21. State Contour Integral.

22. Simple Problems Using Zeroes & Poles.

23. State Cauchy's extended theorem for Multiply Connected Region.

24. What are the formula's to find the residue of a function at multiple pole ?

25. Evaluate ______ where C is the circle |z| = _____.

26.Simple Problems Using "FIND WHICH SINGULARITY IT BELONGS." ?

27. Find the Taylor's series expansion of sin z in about z = pi / 4

28. Find the residue of cot z at the pole z=0.

29.Explain the term singularity.

Note :

- " _____ " implies that model problem will come "
- In Cauchy's Integral Formula ,if a point lies outside the given region , then it's value become zero.

15 Marks :

(You can Omit Atmost one Part Mentioned Below)

PART 1 – Cauchy's Integral Theorem (Problem's + Proof)

PART 2 – Cauchy's Residue Theorem (Problem's + Proof)

PART 3 – Contour Integration. (Problem's + Proof)

PART 4 - Laurent's series (Problem's)

Part 5 – Taylor's Series. (Problem's)

CHAPTER 4 – FOURIER TRANSFORM

2marks :

1. Define Fourier Transform Pair.
2. Define Fourier Cosine Transform Pair.
3. Define Fourier Sine Transform Pair.
4. Define Fourier Integral theorem.
5. Define Finite Fourier Sine & Cosine Transform.
6. State Convolution theorem.
7. State Parseval's identity.
8. State Modulation theorem.
9. Prove that Fourier Transform of an even function f(x) is also an even function.
10. Prove that Fourier Transform of an odd function f(x) is also an odd function.
11. Define Self Reciprocal of Fourier Transform.
12. State Linear property of Fourier Transform.
13. State Shifting property of Fourier Transform.
14. State Change of Scale property of Fourier Transform.

    (STUDY OTHER UNAMED PROPERTY)

15. Define Fourier Transform of derivatives.
16. Simple Problems using F C T & F S T .
17. Simple Problems using Finite F C T & Finite F S T .

    (Note : In Fourier Transform, Property/Theorem is more important)

15 Marks :

1. State and Prove Modulation theorem of Fourier Transform.
2. State and Prove Parseval ' s Identity of Fourier Transform.
3. State and Prove Convolution theorem of Fourier Transform.
4. Problems Using Self Reciprocal.
5. Problems Using Finite Fourier Cosine Transform & Finite Fourier Sine Transform.
6. Problems Using Parseval's Identity.
7. Problems of " Find fourier Transform of f(x) where f(x) = ____ " .

Model Question paper

PART – A

Note :

(i) Answer All the Questions.

(ii) Each Question Carries 2 marks.

1. Find the Laplace Transform of unit step function at t=a.
2. State and prove First Shifting Theorem.
3. Find the inverse Laplace transform of $\frac{1}{(s+1)^4}$.
4. Find $L(\sqrt{t})$ and $L\frac{1}{\sqrt{\pi t}}$.
5. Find $L^{-1}(F(s))$ if $F(s)$ is : $\frac{1}{(s+1)^{\frac{3}{2}}}$

6. If $w = \log z$, then determine where w is non analytic.

7. Show that $v = e^x \cos y$ is harmonic function.

8. Find the invariant points of the transformation $w = \frac{z-1}{z+1}$.

9. State the properties of analytic function.

10. Write C.R equations in Cartesian form and Polar Form.

11. Define Essential Singularity. Give an example.

12. Find the residue of cot z at the pole z=0.

13. State Cauchy's Residue Theorem.

14. Find the Taylor's series expansion of $\frac{1}{z-2}$ in $|z| < 1$.

15. Evaluate $\int_C (z-a)^{-1} dz$ where C is a simple closed curve and the point $z = a$ is (i) outside C, (ii) inside C.

16. Prove that Fourier Transform of an even function f(x) is also an even function.

17. State Shifting property of Fourier Transform.

18. Find the Fourier sine transform of unity.

19. Define Self Reciprocal of Fourier Transform. Give an example .

20. Find the Fourier cosine transform for the function f(x)=sin ax, 0<a<1.

PART – B

Note :

- **Answer any 4 Questions.**

21. (a) (i). Find the Laplace transform of t sint sinh2t and $\dfrac{1-\cos at}{t}$ (8)

(ii). Using convolution theorem, find L^{-1} $\dfrac{1}{(s^2+a^2)^2}$ (7)

(OR)

(b).(i).Find the Laplace transform of the function

$$f(t)=\begin{cases} t, & 0<t<\pi \\ 2\pi-t, & \pi<t<2\pi, \quad f(t+2\pi)=f(t) \end{cases}$$ (7)

(ii).Using Laplace transform technique, solve

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y=e^{-t}\sin t,$$

$$y=0,\ \frac{dy}{dt}=0 \ \ when\ t=0$$ (8)

22.(a) (i). Show that the function $f(z)=\sqrt{|xy|}$ is not analytic at the origin even though C-R equations are satisfied. **(8)**

(ii) Determine the analytic function whose real part is $\frac{\sin 2x}{(\cosh 2y-\cos 2x)}$. **(7)**

(OR)

(b) (i). Find the bilinear transformation that maps the points $1+i, -i, 2-i$ of the z-plane into the points $0, 1, i$ of the w-plane **(8)**

(ii). Discuss the confimal mapping of w=sinz. **(7)**

23.(a). From the integral $\int \frac{dz}{z+2}$, where C is the circle $|z|=1$, find the value of $\int_0^{2\pi} \frac{1+2\cos\theta}{5+4cos\theta} d\theta$ and $\int_0^{2\pi} \frac{\sin\theta}{5+4cos\theta} d\theta$. **(7) + (8)**

(First One Using Cauchy's Integral Theorem & Second One Using Cauchy's Residue Theorem)

(b) (i). Find the Laurent's series of $f(z) = \dfrac{7z-2}{z(z+1)(z+2)}$ in $1 < |z+1| < 3$. (8)

(ii). Evaluate $\int_{\infty}^{\infty} \frac{x^2dx}{(x^2+a^2)(x^2+b^2)}$ using contour integration, where $a > b > 0$. (7)

24.(a) (i) Find the Fourier transform of f(x) = { 1 for |x| < 1

{ 0 for |x| >1 . (8)

(ii) State and Prove Modulation Theroem of Fourier Transform. (7)

(OR)

(b) (i) Verify Parseval's theorem of Fourier Transform for f(x) = $e^{-|x|}$.

(8)

(ii) Find Fourier Sine Transform of f(x) = { sin x ; 0<x<π

{ 0; x> π (7)

ALL THE BEST …!!!!!!!!!!!

“ HARD WORK NEVER FAILS ”